Dynamic Analysis of a Footbridge and Vibration Attenuation Using MTMDs

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Abstract
The design of footbridges is commonly performed considering only the static actions prescribed by standards, with dynamic analysis typically limited to the determination of the structure's natural frequencies. However, evaluating the structural response of footbridges – particularly in terms of acceleration – is crucial to ensure pedestrian comfort during use. In this context, the present study presents a dynamic analysis procedure for a Warren-type truss footbridge subjected to pedestrian-induced dynamic excitation. The results showed that the structural acceleration exceeded the recommended limits, indicating potential discomfort for users. To mitigate this response, Multiple Tuned Mass Dampers (MTMDs) were employed, and their placement was optimized to minimize the structural acceleration. For this purpose, the metaheuristic algorithm One-to-One-Based Optimizer (OOBO) was implemented, demonstrating high efficiency in identifying the optimal solution.

Keywords
Footbridges; Vibrations; Force Model; Dampers; Optimization.

Introduction

The design of footbridges is often guided by static analyses, with dynamic assessments typically limited to verifying that the structure's natural frequencies do not coincide with common excitation frequencies, such as those generated by walking pedestrians. However, this approach has proven inadequate in many situations, particularly when user's comfort is a critical performance criterion. A comprehensive dynamic evaluation, especially with respect to acceleration, is essential for understanding and mitigating potential discomfort caused by pedestrian-induced vibrations.

With the increasing demand for footbridges with longer spans, these structures are becoming more vulnerable to vibrations induced by pedestrian traffic. According to Caetano and Cunha (2013), footbridges with spans of 50 meters or more are often susceptible to vertical vibrations, while those with spans between 80 and 120 meters may experience lateral vibrations, including the possibility of the feedback interaction phenomenon of lock-in. These conditions tend to amplify dynamic responses, frequently resulting in noticeable and uncomfortable vibrations for users. A well-known example is the Millennium Bridge in London, where significant lateral oscillations were caused by the unintentional synchronization of pedestrian footsteps with the structure's natural frequency (Živanović et al., 2005). Although increasing structural stiffness can raise natural frequencies, this typically results in a substantial increase in mass, making the solution impractical.

Given this limitation, passive vibration control devices, such as dynamic dampers, offer a viable and efficient alternative. Among these, the Tuned Mass Damper (TMD), introduced by Frahm in 1909 as a system of mass, spring, and damper, stands out. Den Hartog (1956) later refined the concept by establishing analytical expressions for its optimal design. This concept evolved into Multiple Tuned Mass Dampers (MTMDs), which use several TMDs tuned to target specific vibration modes of the structure, enabling more effective mitigation of dynamic responses.

Several studies have explored strategies to reduce excessive vibrations in footbridges. Miguel and Souza (2023) proposed a robust optimization method using the CIOA algorithm to size TMDs, considering structural uncertainties and different configurations (one, two, or three devices), and compared their results with classical methods by Den Hartog (1956), Warburton (1982), and genetic algorithms. In a similar way, Méndez (2014) assessed the effectiveness of passive multimodal absorbers attached to a fixed beam, employing genetic algorithms and differential evolution to optimize their parameters.

This study proposes a dynamic analysis of a metallic footbridge modeled as a planar Warren truss, whose vibratory response under pedestrian excitation exceeds acceptable acceleration limits reported in the literature. The objective is to investigate the structural response and evaluate the effectiveness of implementing MTMDs to reduce vibration levels without requiring significant changes to the footbridge's mass or stiffness.

Equation of motion

The motion of a system with n degrees of freedom, equipped with j_{tmd} tuned mass dampers and subjected to dynamic pedestrian loading, can be described by the following differential equation:

$$\boldsymbol{M}\ddot{\vec{x}}(t) + \boldsymbol{C}\dot{\vec{x}}(t) + \boldsymbol{K}\vec{x}(t) = \vec{\boldsymbol{F}}(t) \tag{1}$$

Here, $\boldsymbol{M}$, $\boldsymbol{C}$, and $\boldsymbol{K}$ represent the mass, damping, and stiffness matrices, respectively, each of size $(n + j_{tmd}) \times (n + j_{tmd})$. The variable n corresponds to the number of degrees of freedom of the main structure, while j_{tmd} is the number of installed TMDs. The vector $\vec{x}(t)$ contains the displacements and has dimension $(n + j_{tmd})$ and the dot notation indicates time derivatives. The external excitation $\vec{\boldsymbol{F}}(t)$, also of dimension $(n + j_{tmd})$, represents the forces induced by pedestrian movement.

For the damping matrix of the structure, Rayleigh damping is assumed, expressed as $\boldsymbol{C_s} = \alpha_R \boldsymbol{M_s} + \beta_R \boldsymbol{K_s}$, where $\boldsymbol{M_s}$, $\boldsymbol{C_s}$ and $\boldsymbol{K_s}$ are the structure's mass, intrinsic damping, and stiffness matrices (each of size $n \times n$), respectively. The coefficients α_R and β_R are defined as to $\alpha_R = \zeta \frac{2\omega_i \omega_j}{\omega_i + \omega_j}$ and $\beta_R = \zeta \frac{2}{\omega_i + \omega_j}$ where ζ is the desired damping ratio and ω_i, ω_j are selected natural frequencies of the system.

The authors employed a computational routine of Newmark's method to solve the equation of motion.

Human dynamic loading simulation

Several models with varying degrees of complexity have been developed to represent the loads generated by pedestrians crossing footbridges. Notable among them are the single-foot vertical force model proposed by Li et al. (2010), and the more recent and advanced Biodynamic Synchronized Coupled Model (BSCM) introduced by Toso and Gomes (2021), which incorporates pedestrian–structure interaction. Nevertheless, as previously stated, this study adopts the pedestrian load model proposed by Bachmann and Ammann (1987), as described below.

In this study, the excitation caused by normal walking was considered, as it is the most common situation on pedestrian steel footbridges. According to Bachmann and Ammann (1987), the walking speed (v_s) of a pedestrian is related to the step frequency (f_s) and the step length (l_s). The dynamic load shape is defined by Equation 2, where only the first three harmonics are being taken into account.

$$F(t) = G + \Delta G_1 \cdot \sin(2\pi f_s t) + \Delta G_2 \cdot \sin(4\pi f_s t - \varphi_2) + \Delta G_3 \cdot \sin(6\pi f_s t - \varphi_3) \tag{2}$$

in which G is the person's weight, assumed as 800 N; ΔG_1, ΔG_2 and ΔG_3 are the amplitudes of the first, second, and third harmonics, respectively, assumed as $\Delta G_1 = 0.4G$, $\Delta G_2 = 0.1G$, and $\Delta G_3 = 0.1G$; φ_2 and φ_3 are the phase angles of the second and third harmonics relative to the first harmonic, respectively, assumed as $\varphi_2 = \varphi_2 = {}^{\pi}/_{2}$. This vertical load is applied to the truss members accordingly, taking into consideration the pedestrian longitudinal walking speed and proportional to the distance between member nodes.

The above explanations refer to a single person walking on the footbridge. However, in practice, several people often walk on it at the same time. Modeling this situation mathematically is complex. Still, Bachmann and Ammann (1987) suggest using a Poisson distribution to represent the arrival of pedestrians and applying an amplification factor, m, to account for the increased vibration caused by multiple people walking together.

$$m = \sqrt{\lambda T_0} \tag{3}$$

in which λ is the mean flow rate (person/s over the deck for a determined time interval); T_0 is the time required to cross the footbridge of length L_f at velocity v_s ($T_0 = {}^{L_f}/_{v_s}$); and λT_0 represents the number of pedestrians on the footbridge at the same time.

To ensure comfort for pedestrians, Bachamann et al. (1995) stated that an approximate serviceability acceleration limit for footbridges can be obtained by Equation (4).

$$a_{lim} = \min(0.5 {f_1}^{0.5};\ 0.25 {f_1}^{0.78}) \tag{4}$$

where f_1 is the fundamental frequency of the footbridge in Hz.

Tuned Mass Damper (TMD)

The classical formulas proposed by Den Hartog (1956) for undamped structures under harmonic excitations were used to design the TMD, which were positioned according to the optimization results.

$$\mu = \frac{m_d}{M} \tag{5}$$

$$\alpha = \frac{\omega_d}{\omega} = \frac{1}{1+\mu} \tag{6}$$

$$\xi_d = \sqrt{\frac{3\mu}{8(1+\mu)^3}} \tag{7}$$

where μ is the ratio between the mass of the TMD (m_d) and that of the bridge (M); α is the frequency ratio between the TMD (ω_d) and the fundamental frequency of the structure ($\omega = 2\pi f$); ξ_d is the critical damping ratio of the TMD.

Optimization Problem

A commonly adopted performance criterion for evaluating energy dissipation devices, such as Multiple Tuned Mass Dampers (MTMDs), is their effectiveness in reducing the maximum structural displacement. Accordingly, the objective function of the proposed optimization problem is formulated to minimize the expected value of the maximum vertical displacement of the footbridge.

The design variable is the MTMDs location, while its parameters are set using Den Hartog's formulas, as previously explained. The total MTMDs mass is limited to 5% of the structure's total mass. Since this problem can be non-convex and have multiple solutions, it requires optimization methods suited to these challenges. Among the available metaheuristics, the One-to-One-Based Optimizer (OOBO), proposed by Dehghani et al. (2023), has shown good performance and is therefore used in this study. The next section briefly describes the algorithm, with more details found in Dehghani et al. (2023).

One-to-One-Based Optimizer (OOBO)

The optimization problem stated in the previous section can be efficiently solved by using the One-to-One-Based Optimizer (OOBO). The OOBO is a recent metaheuristic optimization algorithm developed by Dehghani *et al.* (2023) and the idea in designing the suggested OOBO is to effectively use the knowledge of all members in the process of updating the algorithm population while preventing the algorithm from relying on specific members of the population. Different of many metaheuristic algorithms, which are strongly dependent on the best member to update the position of population members, the OOBO states that all members of the population should participate in population updating.

In OOBO, to guide the ith member (X_i), a member of the population with position number $k_i(X_{k_i})$ in the population matrix is selected. Based on the values of the objective function of these two members, if the status of member X_{k_i} in the search space is better than that of member X_i, member X_i moves to member X_{k_i}; otherwise, it moves away from member X_i. Based on the above concepts, the process of calculating the new status of population members in the search space is modeled, employing

$$x_{i,d}^{new} = \begin{cases} x_{i,d} + r \cdot \left(x_{k_i,d} - I \cdot x_{i,d}\right), & f_{k_i} < f_i \\ x_{i,d} + r \cdot \left(x_{i,d} - I \cdot x_{k_i,d}\right), & otherwise \end{cases} \qquad (8)$$
$$I = \lfloor 1 + \mathrm{r} \rceil$$

where $x_{i,d}^{new}$ is the new suggested status of the ith member in the dth dimension, $x_{k_i,d}$ is the dimension of the selected member to guide the ith member, $r \in [0,1]$ is a uniformly distributed random variable, f_{k_i} is the objective function value obtained based on X_{k_i}, and the variable I takes values from the set $\{1, 2\}$.

The population updating process in the algorithm is governed by the principle that a candidate solution is accepted only if it yields an improvement in the objective function value. If no improvement is observed, the proposed update is rejected, and the individual retains its current position. This step of the OOBO algorithm can be mathematically formulated as follows:

$$X_i = \begin{cases} X_i^{new}, f_i^{new} < f_i \\ X_i, otherwise, \end{cases} \tag{9}$$

where X_i^{new} is the new suggested status in the search space for the ith population member and f_i^{new} is its value of the objective function.

At this stage of the OOBO algorithm, once the positions of all population members have been updated within the search space, one iteration is completed, and the next iteration begins based on the updated states of the population. This updating procedure, governed by Equations (8)-(9) is repeated until the stopping criterion is met. Upon termination, OOBO returns the best solution found during the search as a quasi-optimal solution to the given problem.

Input: Optimization problem information
Output: Quasi-optimal solution
Start OOBO;
begin
 1. Input optimization problem information;
 2. Set N and T;
 3. Create an initial population matrix;
 4. Evaluate the objective function;
 for $t \leftarrow 1$ **to** T **do**
  5. Update $\mathbf{K}$;
  for $i \leftarrow 1$ **to** N **do**
   6. Calculate X_i^{new} based on Equations (5) and (6);
   7. Compute f_i^{new} based on X_i^{new};
   8. Update X_i using Equation (7);
  end
  9. Save the best solution found so far;
 end
 10. Output the best quasi-optimal solution;
end
End OOBO;

Figure 1 – Pseudocode of OOBO.

Application example

The analyzed footbridge consists of a Warren-type steel truss proposed by Miguel et al. (2013), measuring 39 meters in length and 2.23 meters in height, as illustrated in Figure (2) below. A Young's modulus equal to 200 GPa and a specific mass of 7850 kg/m³ were adopted. The cross-sectional areas of the elements are presented in Table (1) below.

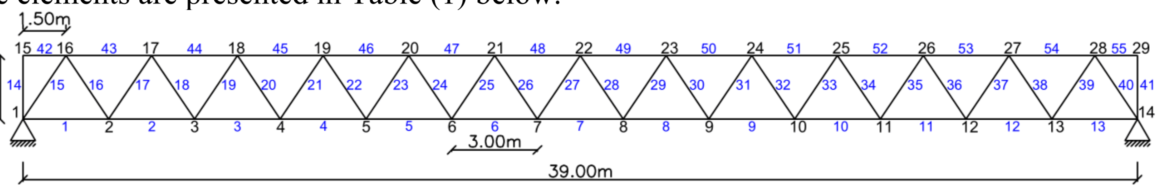


Figure 2 – Warren Truss Footbridge and Possible Locations of Absorbers Devices.

Member number	Area (m²)
1-13	0.0060
14-41	0.0040
42-55	0.0080

Table 1 – Cross sectional areas of the members of the footbridge.

As the objective of this study is to determine the structural response and propose dampers to reduce vibration amplitudes, the first step was to determine the dynamic excitation generated by a pedestrian walking on the footbridge, using the method proposed by Bachman and Amman (1987). For this purpose, a pedestrian with a load of 800 N and a walking frequency of 2 Hz was considered, resulting in the time-dependent load function for a single person, as illustrated in the following figure.

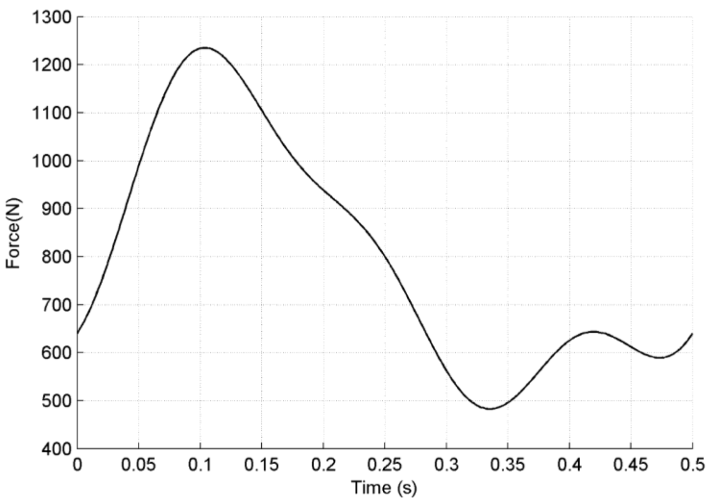
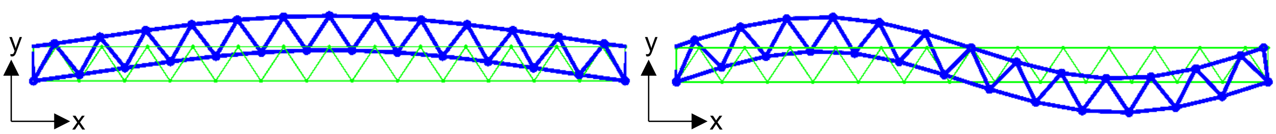


Figure 3 – Load-time Function for Walking of one Person.

However, considering that, in practice, footbridges are used simultaneously by multiple people, Bachman and Amman proposed an amplification factor to represent this collective use. In this study, a pedestrian traffic rate of 0.15 persons/m² was adopted, resulting in an amplification factor of 3.6.

Thus, after determining the dynamic excitation caused by pedestrians, the dynamic analysis of the footbridge is initiated. In the first phase of the study, the vibration modes and natural frequencies of the structure are determined, as illustrated in Figure 4. The fundamental frequency obtained was 5.996 Hz and the second mode frequency was 16.036 Hz, values that match those obtained by Miguel (2013).



(a) Mode 1 – 5.996 Hz (b) Mode 2 – 16.036 Hz

Figure 4 – First Two Mode Shapes of the Studied Footbridge.

Thus, after performing the modal analysis of the structure, the dynamic response of the footbridge due to pedestrian excitation is determined in terms of displacements and accelerations. As illustrated in Figure 5, the maximum vertical displacement at the center of the footbridge (node 22) was about 7 mm, while the calculated RMS displacement was approximately 2.8 mm.

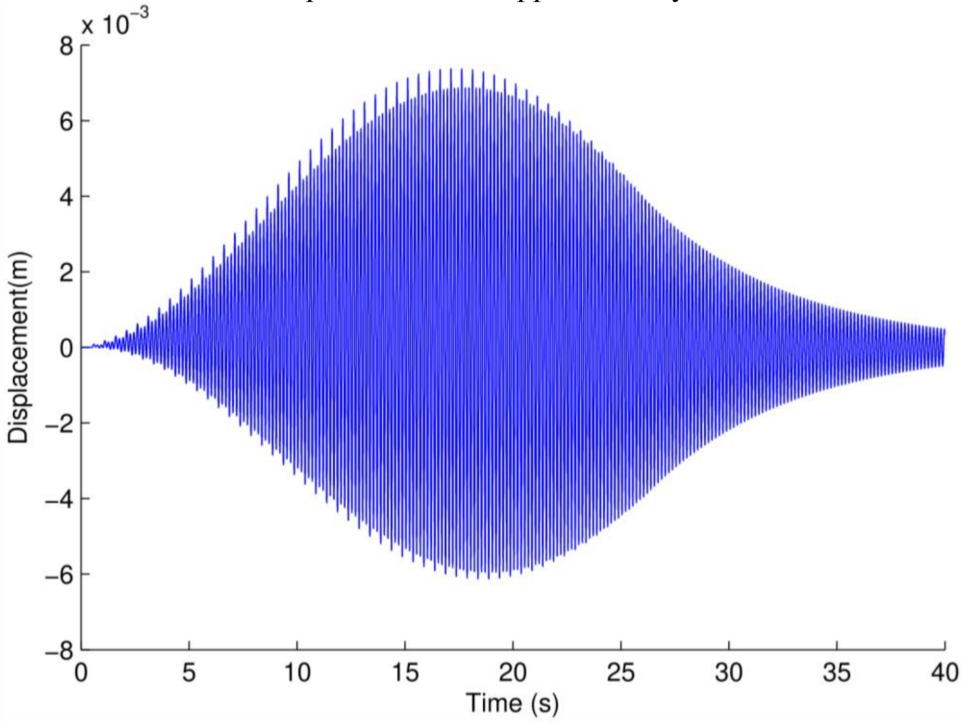


Figure 5 – Vertical Displacement at the Central Node (node 22) of the footbridge.

In terms of acceleration, as illustrated in the figure below, the maximum recorded vertical acceleration was about 10 m/s². Using Equation (4), it is verified that the acceleration limit to ensure user comfort is 1.01 m/s². Therefore, the acceleration observed in the structure exceeds the limit established by Bachman and Amman (1987).

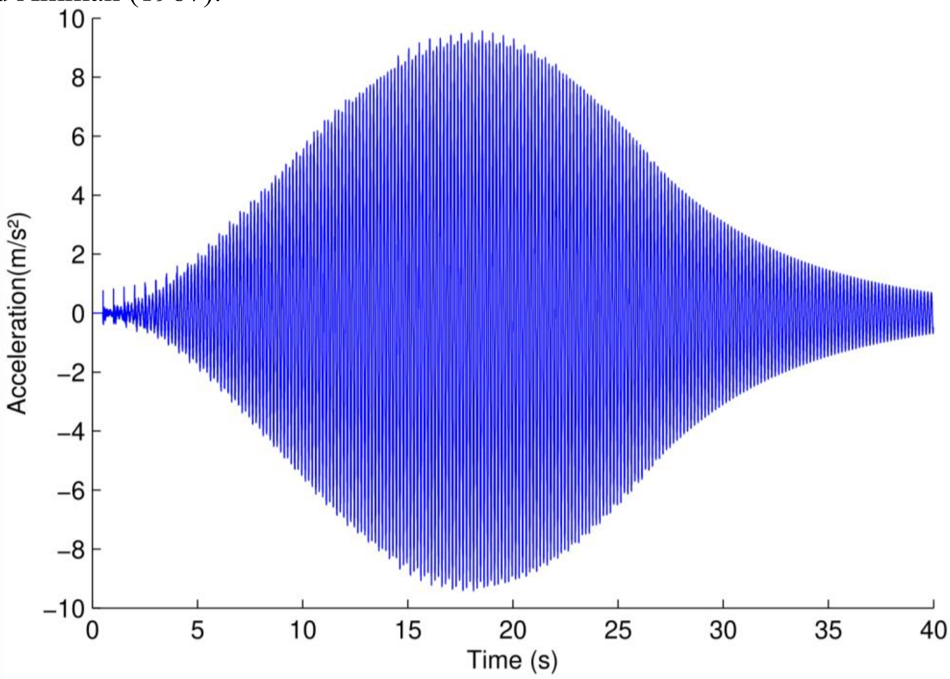


Figure 6 – Vertical Acceleration at the Central Node (node 22) of the Footbridge.

Thus, a solution to mitigate the excessive acceleration consists of implementing MTMDs. However, since the first natural frequency of the structure is not sufficiently close to the excitation frequency to cause resonance, designing MTMDs tuned to the fundamental frequency may not be effective in attenuating the response. To achieve greater efficiency in vibration reduction, the problem was approached by optimizing the placement of the MTMDs on the structure.

For this purpose, the maximum number of MTMDs was limited to three, allowing them to be coupled to any nodes of the structure, including the possibility of placing more than one TMD at the same node. Additionally, the total mass of the TMDs was restricted to 5% of the total mass of the structure, regardless of the number used. As a result, after the optimization process, the best configuration found consisted of three TMDs placed at nodes 7, 8, and 21, as illustrated in Figure 7.

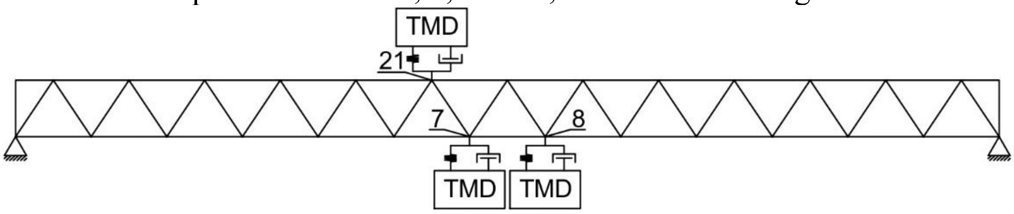


Figure 7 – Placement of MTMDs.

In the following Figure 8, the structural response in terms of displacement can be observed after the implementation of the MTMDs at the nodes indicated in Figure 7. As shown, in terms of maximum displacements, the structure without the dampers exhibited a maximum displacement of about 7 mm. After coupling the MTMDs, the displacements were reduced to approximately 1 mm, representing a reduction of 85.7%. In RMS terms, the displacement decreased from 2.8 mm to 0.39 mm, corresponding to a reduction of 85.93%.

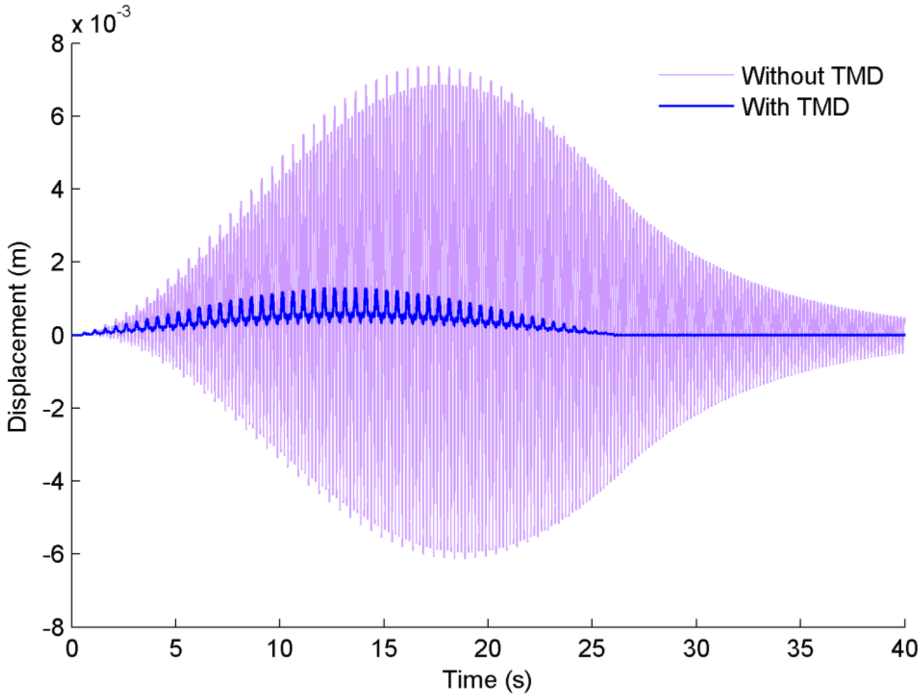


Figure 8 – Vertical Displacement at the Central Node (node 22) of the footbridge.

In the following figure, the structural response in terms of acceleration can be observed after the implementation of the MTMDs at the nodes indicated in Figure 7. As shown, in terms of maximum acceleration, the structure without the dampers exhibited a peak acceleration on the order of 9 m/s². After coupling the MTMDs, the acceleration was reduced to approximately 1 m/s², representing a reduction of 88.89%. In RMS terms, the acceleration decreased from 3.88 m/s² to 0.11 m/s², corresponding to a reduction of 97.07%.

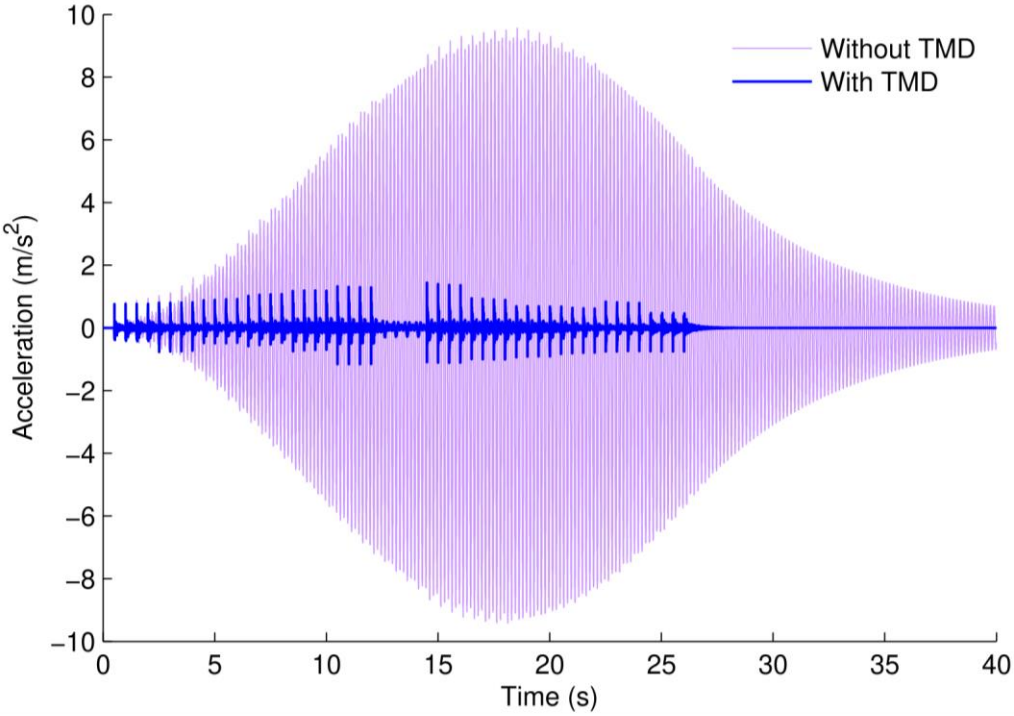


Figure 9 – Vertical Acceleration at the Central Node (node 22) of the footbridge.

Despite the significant reductions in vibration amplitudes, the placement of the TMDs at nodes 7, 8, and 21 is not an intuitive result, since—considering only the first vibration mode—it would be expected that the MTMDs be placed at the central region of the footbridge, where the largest displacements occur. This behavior can be explained by the fact that, although the MTMDs were tuned to the fundamental frequency of the structure, the excitation generated by the pedestrians does not act in that frequency range. Therefore, MTMDs placed at the point of maximum amplitude of the first mode may not perform effectively and, in some cases, may even contribute negatively to the structural response, acting as additional loads that amplify vibrations.

In this context, in order to verify this hypothesis, the pedestrian walking frequency was adjusted to 2.998 Hz, so that the second harmonic of the dynamic excitation—as defined by Equation (2)—would coincide with the fundamental frequency of the structure, thereby inducing the phenomenon of resonance. As a result, after a new optimization process, it was observed that the MTMDs were allocated at nodes 7, 8, and 22, located in the central region of the footbridge, precisely where the largest displacements of the considered vibration mode occur, confirming the previously stated hypothesis. Thus, after the optimization process, the best configuration found consisted of three MTMDs placed at nodes 7, 8, and 22 as illustrated in the following Figure 10.

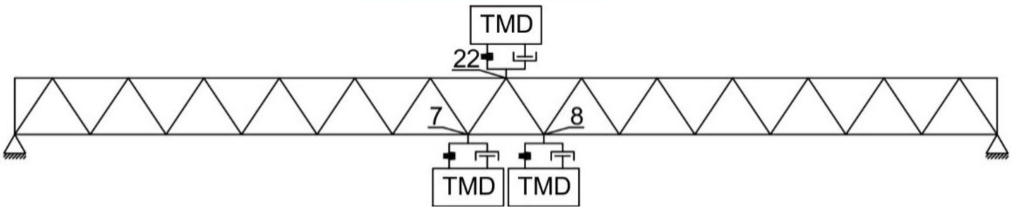


Figure 10 – Placement of MTMDs.

Conclusions

Based on the results obtained, it was found that the structure presented a fundamental frequency of 5.996 Hz, while the excitation frequency induced by pedestrian walking was 2 Hz. Although these frequencies are not close, the structure, due to its large span, exhibited acceleration responses that exceeded the limits recommended in the literature, which justifies the implementation of MTMDs as a strategy to reduce vibration amplitudes.

The optimization of the MTMDs placement on the structure indicated that, although they were tuned to the first mode of vibration, the devices were not coupled exactly at the mid-span, as would be expected based on the modal shape, due to the excitation acting outside the range of the structure's fundamental frequency. Nevertheless, a significant reduction in the structural response was observed, both in terms of displacement and acceleration. Concerning the initial more realistic case (no resonance), the RMS displacement was reduced by approximately 85.93%, while the RMS acceleration showed a reduction of about 97.07%, meeting the acceleration limit proposed by Bachmann et al. (1995) and demonstrating the effectiveness of the MTMD system in mitigating pedestrian-induced vibrations.

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